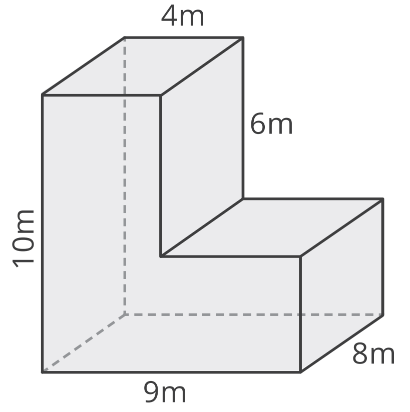
# Calculating Surface Areas and Volumes of Composite Solids
## Answers

1. Find the volume and surface area of each solid.

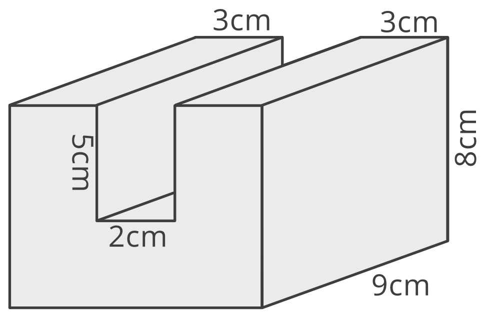
a.




**Volume: 480m$^3$**

**Surface area: 424m$^2$**

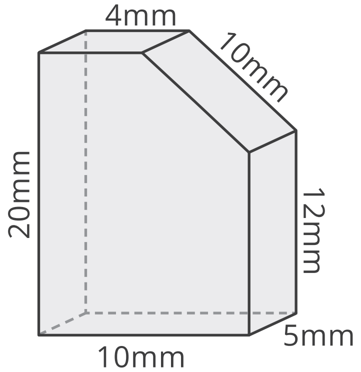
b.




**Volume: 486cm$^3$**

**Surface area: 486cm$^2$**

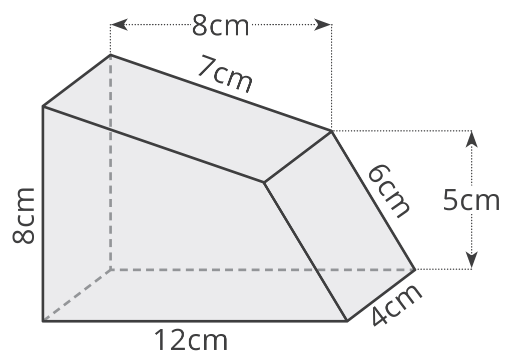
c.




**Volume: 880mm$^3$**

**Surface area: 632mm$^2$**

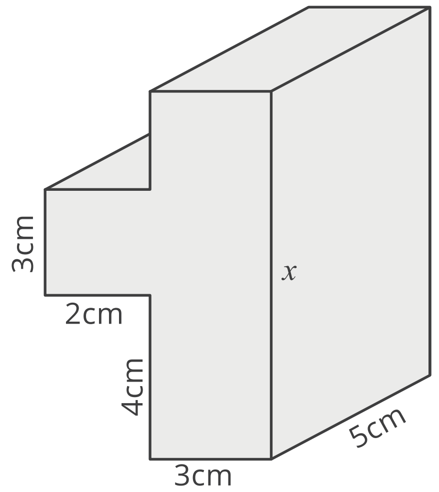
d.




**Volume: 248cm$^3$**

**Surface area: 256cm$^2$**

2. The solid below has a volume of $165\text{cm}^3$. Find the height, $x$, of the solid.



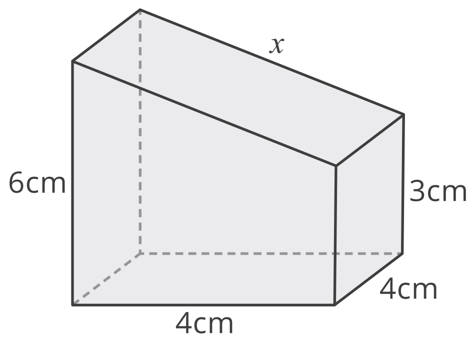

**Area of front face: 165 ÷ 5 = 33cm$^2$**

**Subtract 3 × 2 rectangle: 30 – 6 = 27cm$^2$**

**Height of remaining rectangle: 27 ÷ 3 = 9cm**

**$x$ = 9cm (alternate methods will result in the same answer)**

3. The surface area of the solid below is $108\text{cm}^2$. Find the length of side $x$.

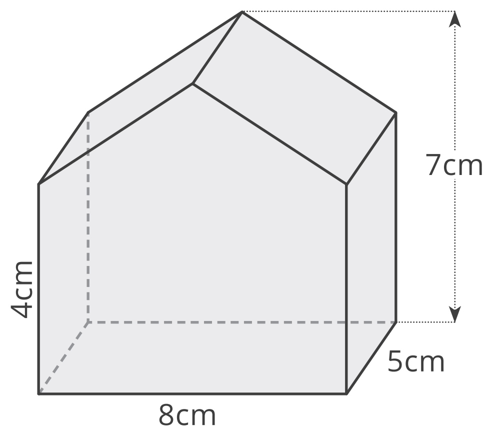



**Surface area of known faces: 88cm$^2$**

**Area of top: 108 – 88 = 20cm$^2$**

**$x$ = 20 ÷ 4 = 5cm**

4. The solid below is formed of an isosceles triangular prism placed on a rectangular prism. Find its surface area.




**Front and back: 44cm$^2$**

**Left and right: 20cm$^2$**

**Base: 40cm$^2$**

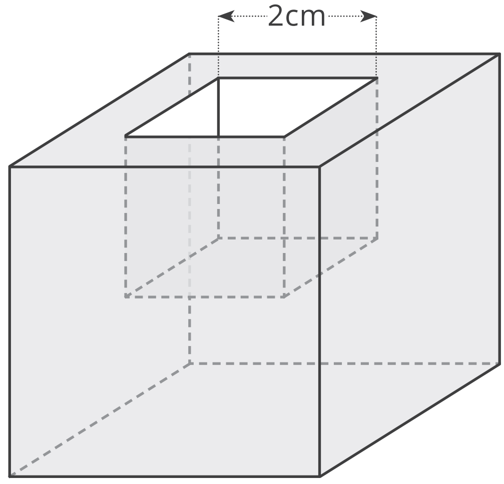
**Top: Using Pythagoras's theorem to find the diagonal: $\sqrt{3^2 + 4^2}$ = 5cm**

**5 × 5 = 25cm$^2$**

**Surface area:**

**44 + 44 + 20 + 20 + 40 + 25 + 25 = 218cm$^2$**

5. The solid below is made up a cube removed from a larger cube. Find the volume and surface area of the shape.




**Volume: 5 × 5 × 5 – 2 × 2 × 2 = 117cm$^3$**

**Surface area:**

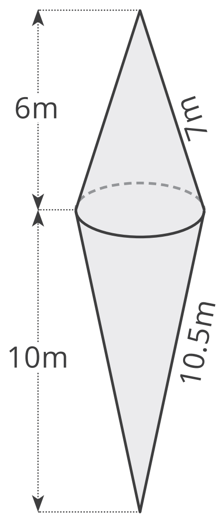
**25 + 25 + 25 + 25 + 25 + 21 + 4 + 4 + 4 + 4 + 4 = 166cm$^2$**

The following formulae are provided in GCSE exams, you may find them helpful for questions 6-8.

- Curved surface area of a cone = $\pi rl$
- Volume of a cone = $\frac{1}{3}\pi r^2 h$
- Volume of a pyramid = $\frac{1}{3}$ area of base × height (given in OCR exams only)
- Surface are of a sphere = $4\pi r^2$
- Volume of a sphere = $\frac{4}{3}\pi r^3$

6. Find the volume and surface area of each shape. Where possible, give your answers in terms of π.

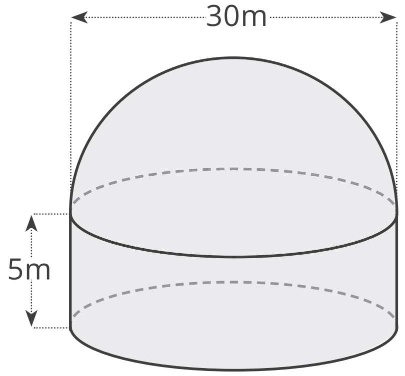
a. The shape below is made up of two cones. Each cone has a diameter of 6cm.




**Volume: 30π + 18π = 48π m$^3$**

**Surface area: 31.5π + 21π = 52.5π m$^2$**

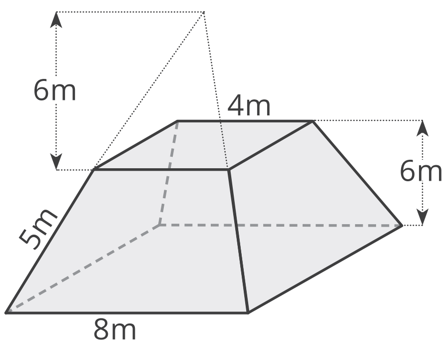
b. The solid below is made up of a cylinder topped with a hemisphere.




**Volume: 1125π + 2250π = 3375π m$^3$**

**Surface area: 225π + 150π + 450π = 825π m$^2$**

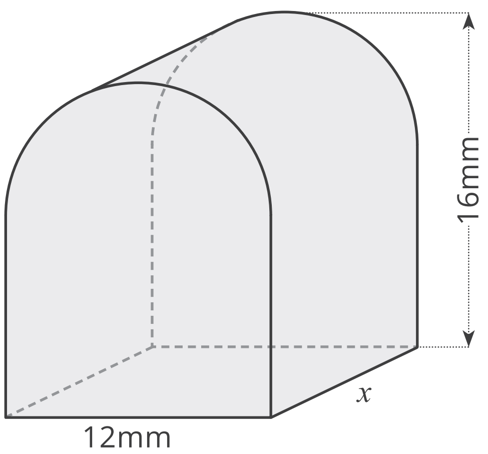
c. The is a square-based pyramid, with the top 6m removed.




**Volume: 256 – 32 = 224m$^3$**

**Surface area:**
**30 + 30 + 30 + 30 + 16 + 64 = 200cm$^2$**

7. The solid below is made up of a semi-circular prism placed on top of a rectangular prism. If its volume is (600 + 90π) mm$^2$, what is the length of side $x$?



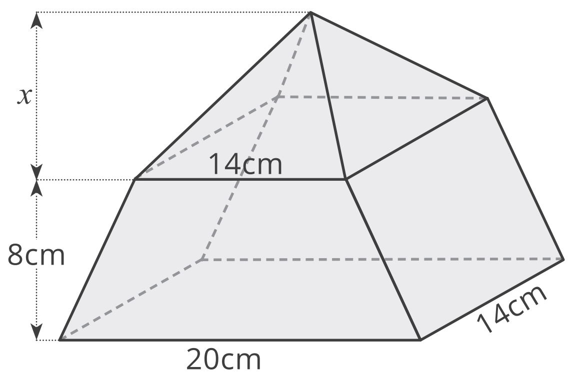

**The diameter of the semi-circle is 12mm, therefore its radius is 6mm. This is also the height of the semi-circle, so the height of the rectangle is 10mm.**

**The cross-sectional area is therefore:**
**$10 \times 12 + \frac{1}{2} \times 36 \times \pi$, or $120 + 18\pi$.**

**600 ÷ 120 = 5; 90π ÷ 18π = 5.**

**$x$ = 5**

8. The solid below is made up of a square-based pyramid placed on top of a trapezoid prism. The volume of the solid is 2296cm$^3$. Find the length of side $x$.




**The cross-sectional area of the prism is 136cm$^2$.**

**The volume of the prism is 1904cm$^3$.**

**The volume of the pyramid is 2296 – 1904 = 392cm$^3$.**

**For the pyramid:**

$V = \frac{1}{3} \times 14 \times 14 \times x$

$392 = \frac{1}{3} \times 196 \times x$

$1176 = 196x$

$x = 6\text{cm}$

**Challenge**

A machine part is made by taking a cube of metal of volume 512cm$^3$ and removing a hemisphere from one face. The hemisphere has a diameter equal to the width of the cube.

The company is trying to reduce waste.

a. How many parts would they need to make for the waste material to be equivalent to one complete part?
b. The company does not want to change their manufacturing process. Using their current method, how many complete parts would they need to make for the waste to be sufficient to manufacture one new part?

**a. The width of the cube is: $\sqrt[3]{512}$ = 8cm**

**The volume of the hemisphere removed is: $\frac{1}{2} \times \frac{4}{3} \times \pi \times 4^3 = 134.04\text{cm}^3$**

**The volume of the completed part is: 512 – 134.04 = 377.96cm$^3$**

**377.96 ÷ 134.04 = 2.8, therefore you would need to make 3 complete parts for the waste material to be equivalent to one new part.**

**b. Their manufacturing process starts with a cube with volume 512cm$^3$.**

**512 ÷ 134.04 = 3.8. They would need to make 4 complete parts.**

# Calculating Surface Areas and Volumes of Composite Solids

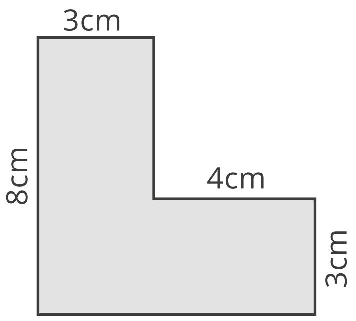
**Prior learning for example 1 and questions 1 to 5:**

- Finding the area of a composite shape.
- Finding the volume of a prism.
- Finding the surface area of a prism.
- Finding a missing side using Pythagoras' theorem.

**Additional prior learning for example 2 and questions 6 to 9:**

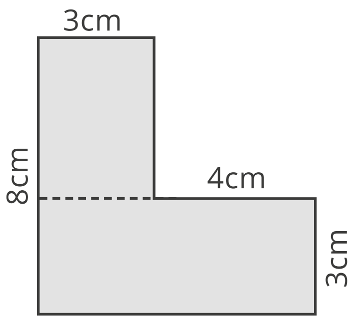
- Finding the volume of a sphere, cone or pyramid.
- Finding the surface area of a sphere, cone or pyramid.
- Answering in terms of $\pi$.

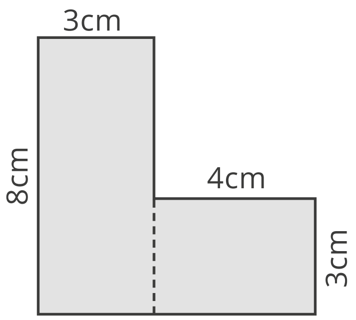
A composite shape is made up of more than one shape. To find the area, we usually consider the simpler shapes that make it up. Consider the L-shape below:

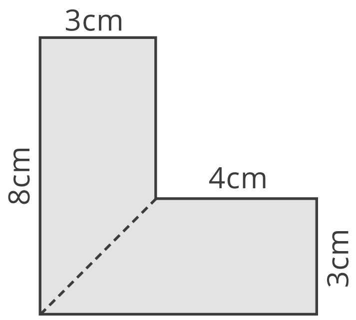



We can't find the area of the L-shape by substituting all the lengths into a formula. Instead, you divide it into simpler shapes that you can find the area of.

In this case, you could divide it into a small rectangle on top of a larger rectangle, a large rectangle next to a smaller rectangle, or even two trapeziums:



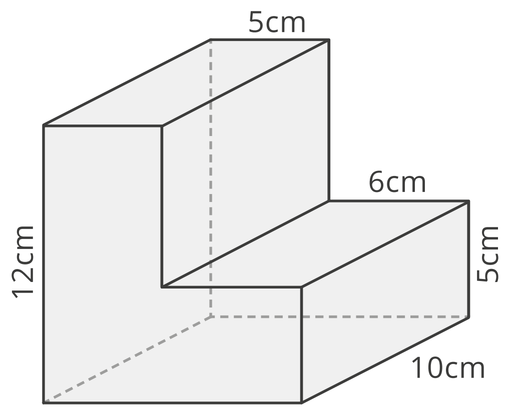




Each method would give the same result for the area. There is no 'right' way to split up the shape; just divide it in a way that you think will make the calculations easier.

A composite solid is similar to a composite shape, but in three dimensions. We will approach finding the volume and surface area in a similar way to find the area of a composite shapes – by splitting it into simpler solids. These questions can be more difficult because they rely not just on maths, but also on your ability to imagine and manipulate 3D objects in your head.

**Example 1:** Find the volume and surface area of the prism below.

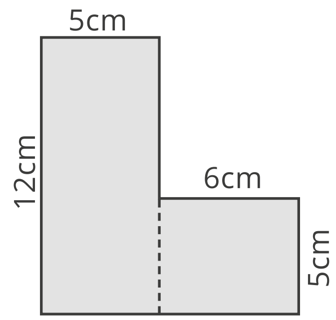



We'll start with the volume. There are two ways to consider this problem. We can treat the solid as a single prism, find the area of the front face then multiply by the depth to find the volume.

Alternatively, we can treat it as two separate prisms (for example, a large cuboid next to a smaller cuboid), find the volume of each prism and add them to find the volume of the solid.

While this affects the order of some of our calculations, it will not affect the result or the overall difficulty, so choose the method that makes more sense to you. In this case, we will treat it as a single prism.

A prism has a consistent cross-section – to find the volume of a prism, we first need to find the area of that cross-section. For this solid, that's the area of the front face.




We can treat this as two rectangles. The left rectangle has height 12cm and width 5cm, while the right rectangle has height 5cm and width 6cm:

Left rectangle area: $12 \times 5 = 60\text{cm}^2$

Right rectangle area: $6 \times 5 = 30\text{cm}^2$

Total area: $60 + 30 = 90\text{cm}^2$

To find the volume, we multiply the cross-sectional area ($90\text{cm}^2$) by the depth (10cm):

**Volume = $90 \times 10 = 900\text{cm}^3$**

Now, we need find the surface area. When finding the surface are of any 3D solid, particularly a composite solid, the most important thing is to make sure you find the area of each face.

Be careful not to miss any or count any more than once.

There are two ways to do this. Again, which you find easier will depend on how you visualise the shapes.

Firstly, we could imagine how the shape would look in 3D, list the faces, then find the area of each face. Secondly, we could 'unfold' the shape to draw the net, then use that as a guide to find the areas. In both cases, we would then add up the areas to find the total surface area. We'll try both methods.

First, we'll list the faces. To avoid confusion, make sure you're clearly identifying each face:

*Base:* $11 \times 10 = 110\text{cm}^2$ (the width of the base is $5 + 6 = 11\text{cm}$)

*Top:* $5 \times 10 = 50\text{cm}^2$

*Left:* $12 \times 10 = 120\text{cm}^2$

*Bottom Right:* $5 \times 10 = 50\text{cm}^2$

*Middle Right:* $6 \times 10 = 60\text{cm}^2$

*Top Right:* $7 \times 10 = 70\text{cm}^2$ (the height of the top right face is $12 - 5 = 7\text{cm}$)

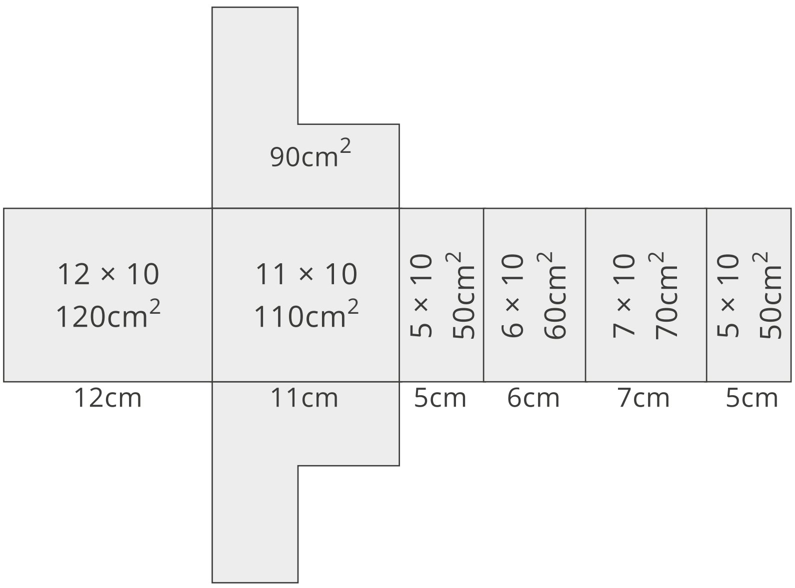
*Front:* $90\text{cm}^2$ (we've already worked this out for the volume)

*Back:* $90\text{cm}^2$

**Total surface area: $110 + 50 + 120 + 50 + 60 + 70 + 90 + 90 = 640\text{cm}^2$**

Alternatively, we can sketch out the net and fill in the areas of each face:

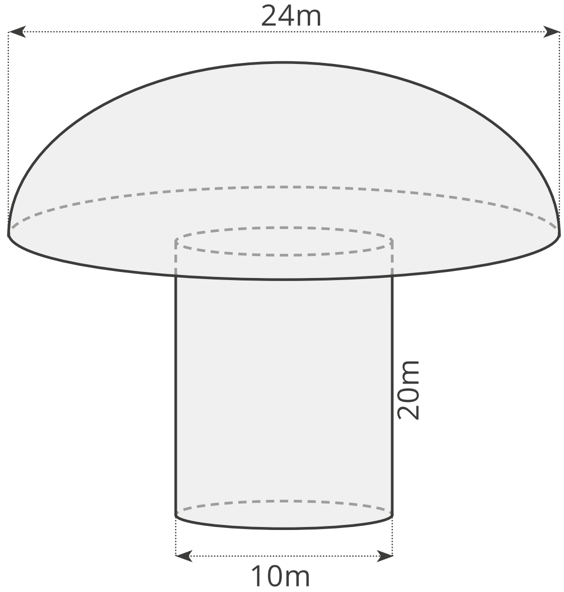
**Total surface area: $110 + 50 + 120 + 50 + 60 + 70 + 90 + 90 = 640\text{cm}^2$**




As you can see, the results of the two methods are exactly the same – you can use whichever you prefer.

**Example 2:** The building below is made up of a hemisphere on top of a cylinder. The diameter of the hemisphere is 24m, the height of the cylinder is 20m and the diameter of the cylinder is 10m.

Find the volume and surface area of the building. Give your answer in terms of π.




Although this question is more challenging, we will approach it using the same processes. Firstly, we'll find the volume. In this case, our composite solid is not a prism, so we'll split the shape into two solids, a cylinder and a hemisphere, then find the volume of each.

A cylinder is a circular prism. As such, its volume is the cross-sectional area multiplied by the depth. Remember, when finding the area of a circle we need the radius (5m), not the diameter (10m).

Volume of the cylinder: $\pi r^2 \times h = 5^2 \times 20 \times \pi = 500\pi$ m$^3$

The question specifies that our answer should be in terms of π, so we will leave the answer as it is.

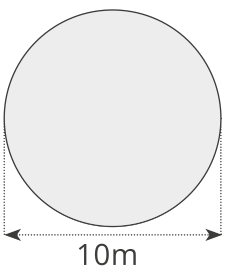
A hemisphere is half a sphere; we will use the formula for the volume of a sphere ($V = \frac{4}{3}\pi r^3$) and multiply it by $\frac{1}{2}$:

Volume of the hemisphere: $\frac{1}{2} \times \frac{4}{3} \times \pi \times r^3 = \frac{4}{6} \times 12^3 = 1152\pi$ m$^3$

**Total volume: 500π + 1152π = 1652π m$^3$**

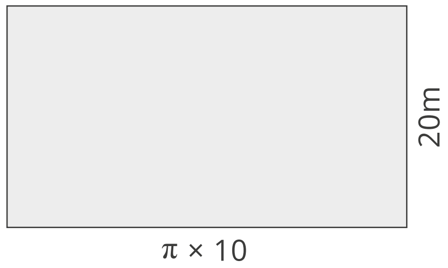
For the surface area, we again have a choice between labelling the faces or using a net. In this case, drawing a clear net would be challenging, so we will label the faces.

Let's consider the cylinder first. The base of the cylinder is simply a circle:




Base of the cylinder: $\pi r^2 = \pi \times 5^2 = 25\pi$ m$^2$

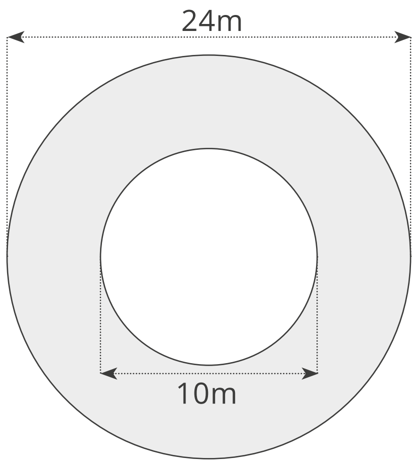
The curved face of the cylinder is a rectangle. Its height is simply the height of the cylinder and its width is the circumference of the circle on the base of the cylinder.




Curved face of cylinder: $h \times \pi d = 20 \times \pi \times 10 = 200\pi\ \text{m}^2$

Next, let's consider the hemisphere. Remember, a hemisphere is just half a sphere.

The flat base of the hemisphere is not a full circle, but a donut shape, with the top of the cylinder cut out:




Flat base of the hemisphere: $\pi \times 12^2 - \pi \times 5^2 = 119\pi\ \text{m}^2$

For the curved face, we'll use the formula for the surface area of a sphere ($A = 4\pi r^2$) and multiply if by $\frac{1}{2}$. We can't easily draw the surface of the hemisphere, but we know the radius is 12m.

Curved outside of the hemisphere: $\frac{1}{2} \times 4\pi \times r^2 = \frac{1}{2} \times 4 \times 12^2 \times \pi = 288\pi\ \text{m}^2$

Finally, we'll add up each face:

**Surface area = $25\pi + 119\pi + 200\pi + 288\pi = 632\pi\ \text{m}^2$**

Although the shape for this example was more complex, the process was the same. Break the solid down into clearly identified pieces (either simpler solids or faces), find the volume or surface area of each piece, then add to find the total volume or surface area.

## Your Turn

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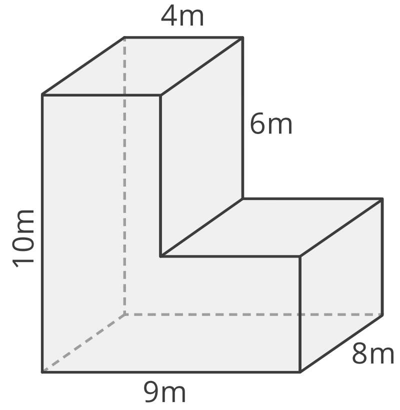


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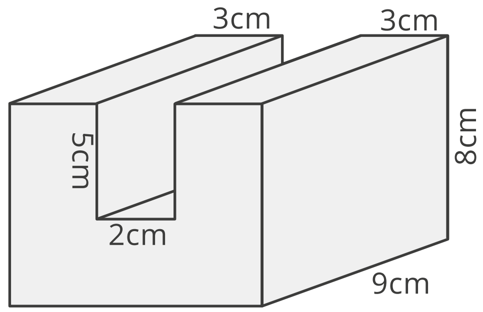


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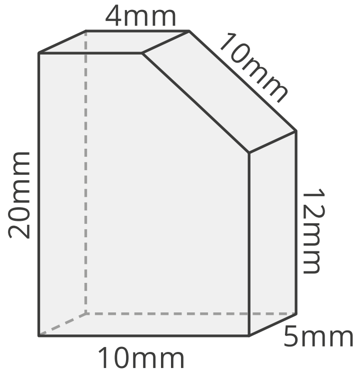


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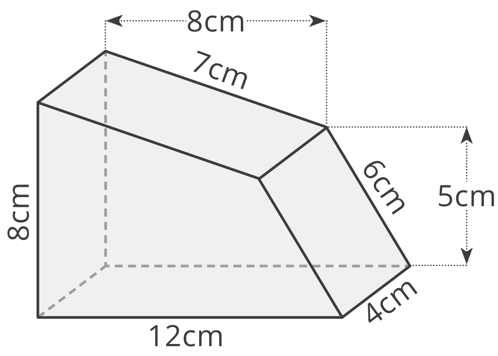

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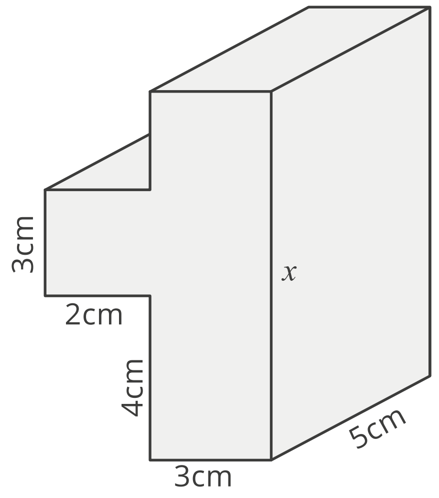


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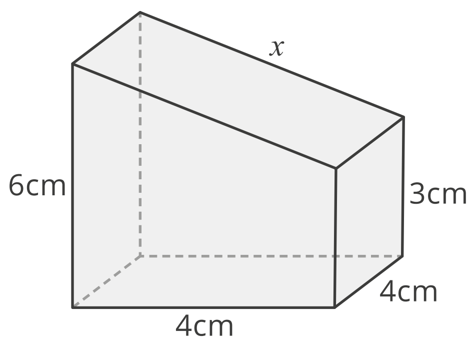


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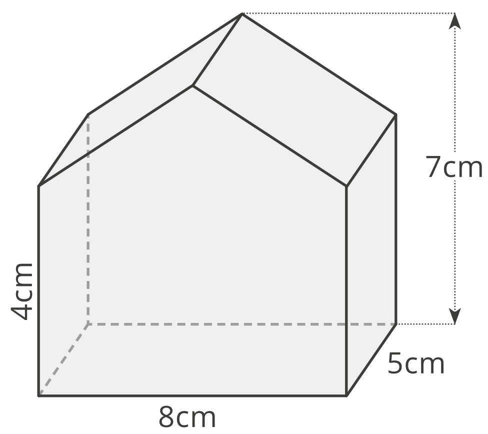

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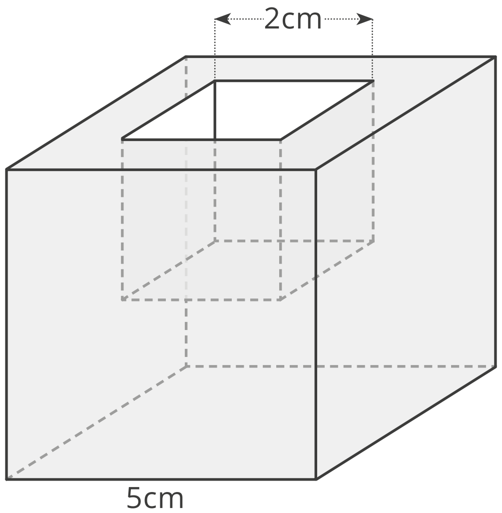


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- Volume of a sphere = $\frac{4}{3}\pi r^3$

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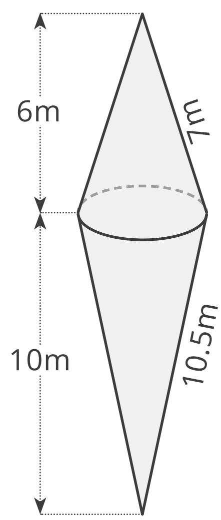

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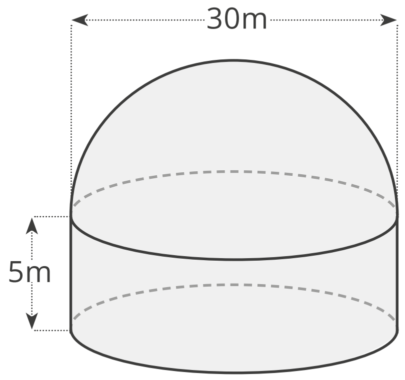


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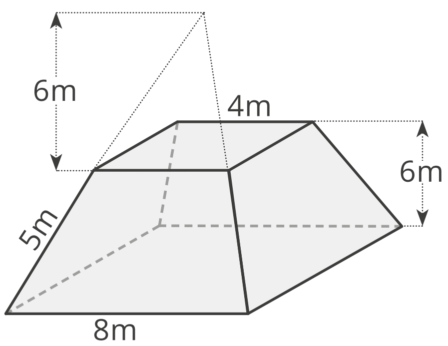


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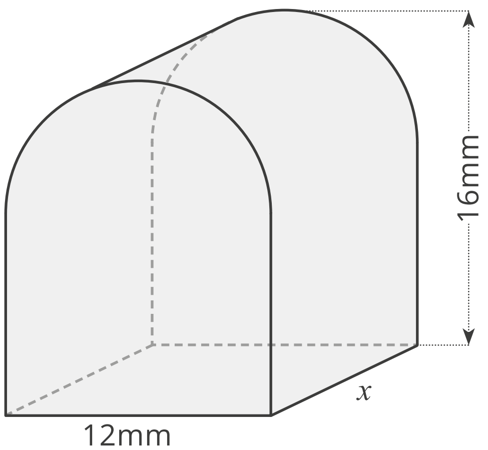

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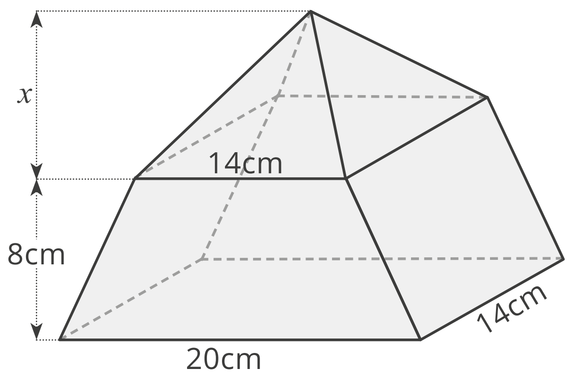


## Challenge

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- Finding the volume of a prism.
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- Finding a missing side using Pythagoras' theorem.

**Additional prior learning for example 2 and questions 6 to 9:**

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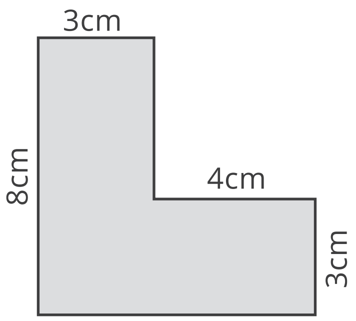
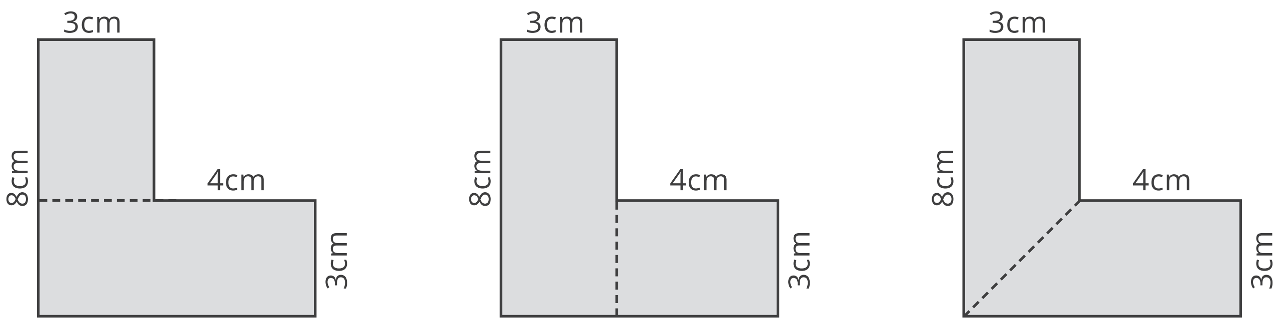
- Finding the surface area of a sphere, cone or pyramid.
- Answering in terms of $\pi$.

A composite shape is made up of more than one shape. To find the area, we usually consider the simpler shapes that make it up. Consider the L-shape below:




We can't find the area of the L-shape by substituting all the lengths into a formula. Instead, you divide it into simpler shapes that you can find the area of.

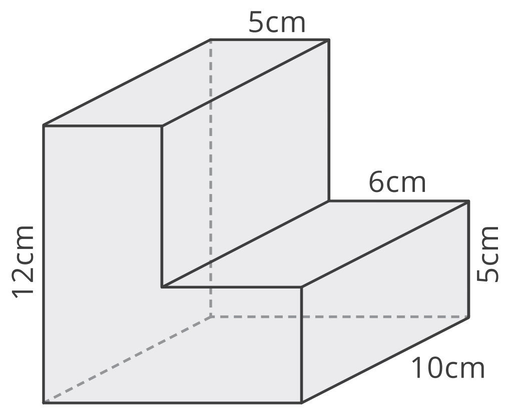
In this case, you could divide it into a small rectangle on top of a larger rectangle, a large rectangle next to a smaller rectangle, or even two trapeziums:




Each method would give the same result for the area. There is no 'right' way to split up the shape; just divide it in a way that you think will make the calculations easier.

A composite solid is similar to a composite shape, but in three dimensions. We will approach finding the volume and surface area in a similar way to find the area of a composite shapes – by splitting it into simpler solids. These questions can be more difficult because they rely not just on maths, but also on your ability to imagine and manipulate 3D objects in your head.

**Example 1:** Find the volume and surface area of the prism below.

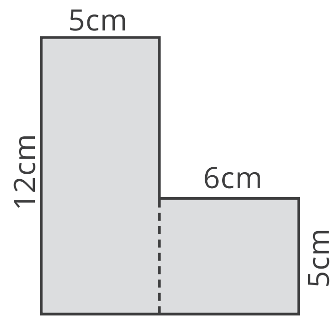



We'll start with the volume. There are two ways to consider this problem. We can treat the solid as a single prism, find the area of the front face then multiply by the depth to find the volume.

Alternatively, we can treat it as two separate prisms (for example, a large cuboid next to a smaller cuboid), find the volume of each prism and add them to find the volume of the solid.

While this affects the order of some of our calculations, it will not affect the result or the overall difficulty, so choose the method that makes more sense to you. In this case, we will treat it as a single prism.

A prism has a consistent cross-section – to find the volume of a prism, we first need to find the area of that cross-section. For this solid, that's the area of the front face.




We can treat this as two rectangles. The left rectangle has height 12cm and width 5cm, while the right rectangle has height 5cm and width 6cm:

Left rectangle area: $12 \times 5 = 60\text{cm}^2$

Right rectangle area: $6 \times 5 = 30\text{cm}^2$

Total area: $60 + 30 = 90\text{cm}^2$

To find the volume, we multiply the cross-sectional area ($90\text{cm}^2$) by the depth (10cm):

**Volume = $90 \times 10 = 900\text{cm}^3$**

Now, we need find the surface area. When finding the surface are of any 3D solid, particularly a composite solid, the most important thing is to make sure you find the area of each face.

Be careful not to miss any or count any more than once.

There are two ways to do this. Again, which you find easier will depend on how you visualise the shapes.

Firstly, we could imagine how the shape would look in 3D, list the faces, then find the area of each face. Secondly, we could 'unfold' the shape to draw the net, then use that as a guide to find the areas. In both cases, we would then add up the areas to find the total surface area. We'll try both methods.

First, we'll list the faces. To avoid confusion, make sure you're clearly identifying each face:

*Base:* $11 \times 10 = 110\text{cm}^2$ (the width of the base is $5 + 6 = 11\text{cm}$)

*Top:* $5 \times 10 = 50\text{cm}^2$

*Left:* $12 \times 10 = 120\text{cm}^2$

*Bottom Right:* $5 \times 10 = 50\text{cm}^2$

*Middle Right:* $6 \times 10 = 60\text{cm}^2$

*Top Right:* $7 \times 10 = 70\text{cm}^2$ (the height of the top right face is $12 – 5 = 7\text{cm}$)

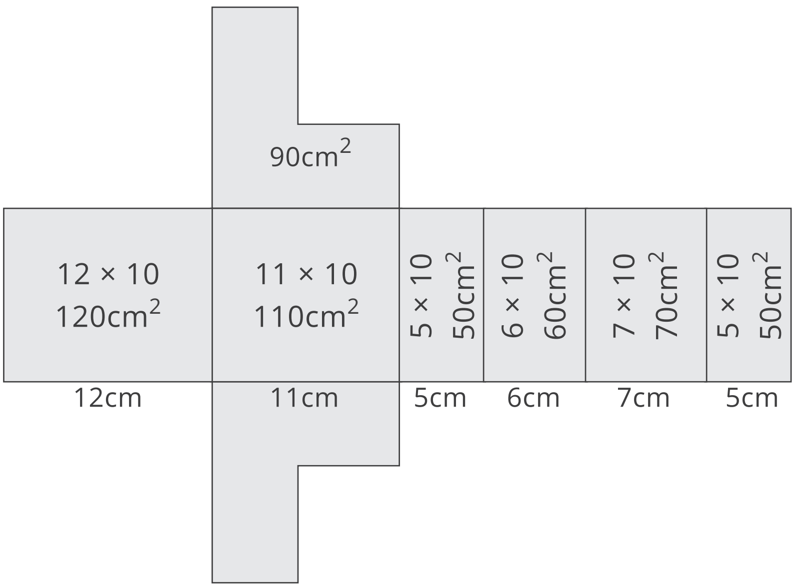
*Front:* $90\text{cm}^2$ (we've already worked this out for the volume)

*Back:* $90\text{cm}^2$

**Total surface area: $110 + 50 + 120 + 50 + 60 + 70 + 90 + 90 = 640\text{cm}^2$**

Alternatively, we can sketch out the net and fill in the areas of each face:

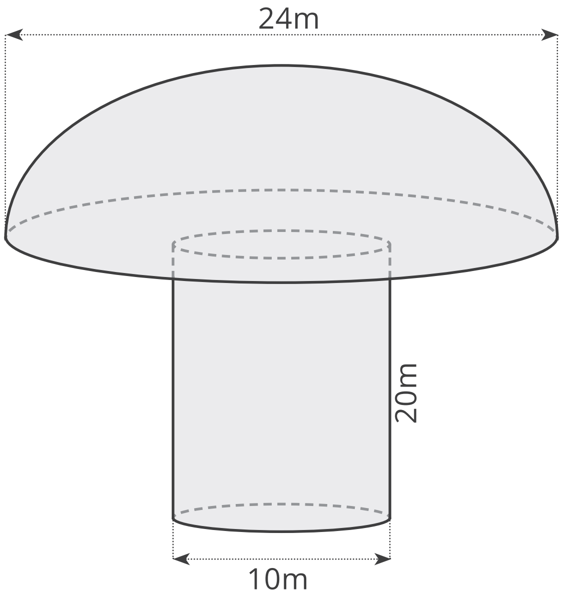
**Total surface area: $110 + 50 + 120 + 50 + 60 + 70 + 90 + 90 = 640\text{cm}^2$**




As you can see, the results of the two methods are exactly the same – you can use whichever you prefer.

**Example 2:** The building below is made up of a hemisphere on top of a cylinder. The diameter of the hemisphere is 24m, the height of the cylinder is 20m and the diameter of the cylinder is 10m.

Find the volume and surface area of the building. Give your answer in terms of π.




Although this question is more challenging, we will approach it using the same processes. Firstly, we'll find the volume. In this case, our composite solid is not a prism, so we'll split the shape into two solids, a cylinder and a hemisphere, then find the volume of each.

A cylinder is a circular prism. As such, its volume is the cross-sectional area multiplied by the depth. Remember, when finding the area of a circle we need the radius (5m), not the diameter (10m).

Volume of the cylinder: $\pi r^2 \times h = 5^2 \times 20 \times \pi = 500\pi$ m$^3$

The question specifies that our answer should be in terms of π, so we will leave the answer as it is.

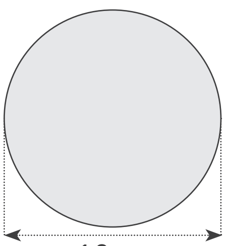
A hemisphere is half a sphere; we will use the formula for the volume of a sphere ($V = \frac{4}{3}\pi r^3$) and multiply it by $\frac{1}{2}$:

Volume of the hemisphere: $\frac{1}{2} \times \frac{4}{3} \times \pi \times r^3 = \frac{4}{6} \times 12^3 = 1152\pi$ m$^3$

**Total volume: $500\pi + 1152\pi = 1652\pi$ m$^3$**

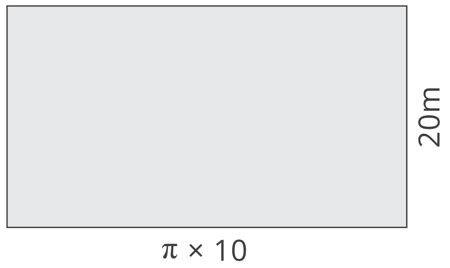
For the surface area, we again have a choice between labelling the faces or using a net. In this case, drawing a clear net would be challenging, so we will label the faces.

Let's consider the cylinder first. The base of the cylinder is simply a circle:




Base of the cylinder: $\pi r^2 = \pi \times 5^2 = 25\pi$ m$^2$

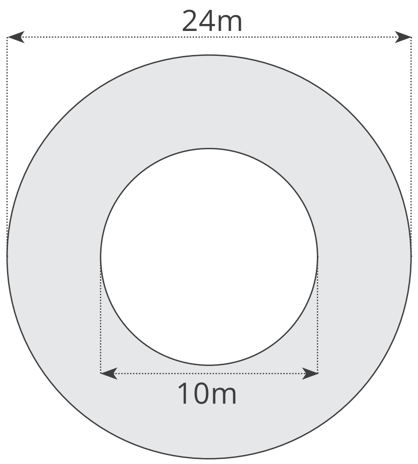
The curved face of the cylinder is a rectangle. Its height is simply the height of the cylinder and its width is the circumference of the circle on the base of the cylinder.




Curved face of cylinder: $h \times \pi d = 20 \times \pi \times 10 = 200\pi\ \text{m}^2$

Next, let's consider the hemisphere. Remember, a hemisphere is just half a sphere.

The flat base of the hemisphere is not a full circle, but a donut shape, with the top of the cylinder cut out:




Flat base of the hemisphere: $\pi \times 12^2 - \pi \times 5^2 = 119\pi\ \text{m}^2$

For the curved face, we'll use the formula for the surface area of a sphere ($A = 4\pi r^2$) and multiply if by $\frac{1}{2}$. We can't easily draw the surface of the hemisphere, but we know the radius is 12m.

Curved outside of the hemisphere: $\frac{1}{2} \times 4\pi \times r^2 = \frac{1}{2} \times 4 \times 12^2 \times \pi = 288\pi\ \text{m}^2$

Finally, we'll add up each face:

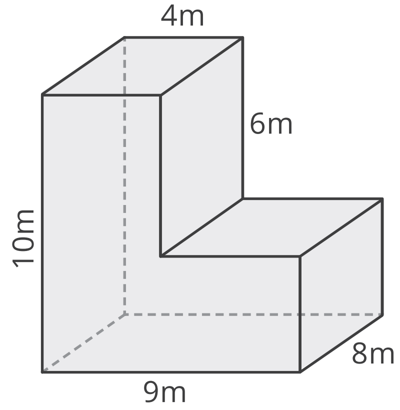
**Surface area = $25\pi + 119\pi + 200\pi + 288\pi = 632\pi\ \text{m}^2$**

Although the shape for this example was more complex, the process was the same. Break the solid down into clearly identified pieces (either simpler solids or faces), find the volume or surface area of each piece, then add to find the total volume or surface area.

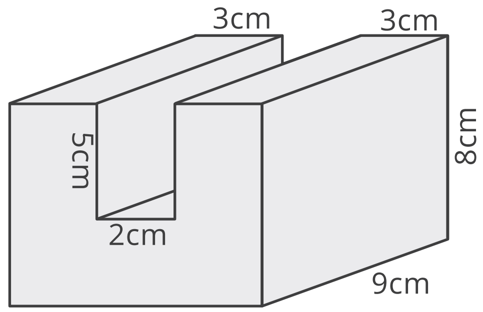
## Your Turn

1. Find the volume and surface area of each solid.

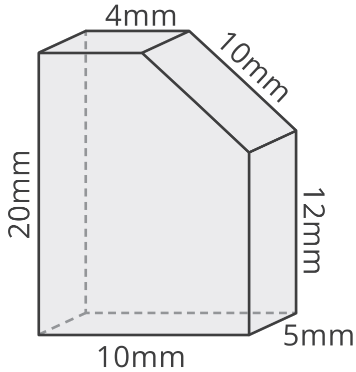
a.



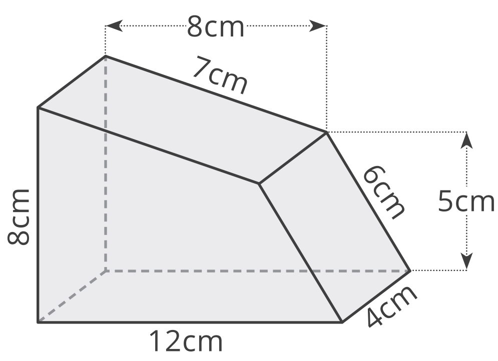

b.



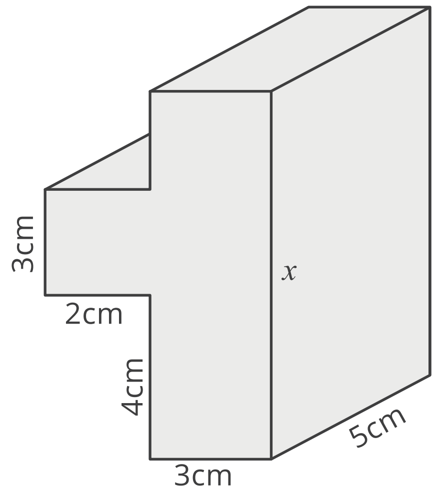

c.



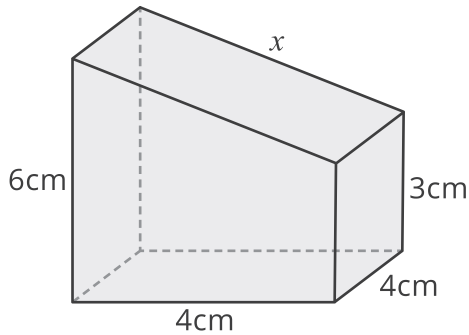

d.



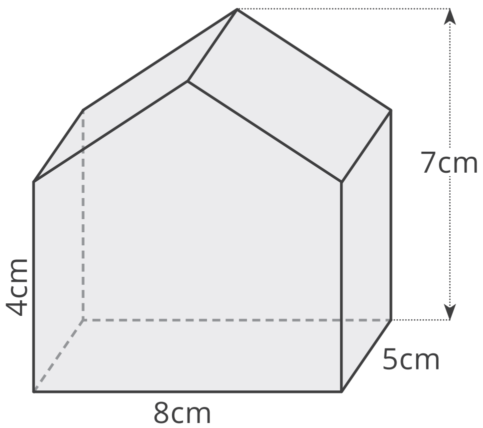
2. The solid below has a volume of 165cm$^3$. Find the height, $x$, of the solid.



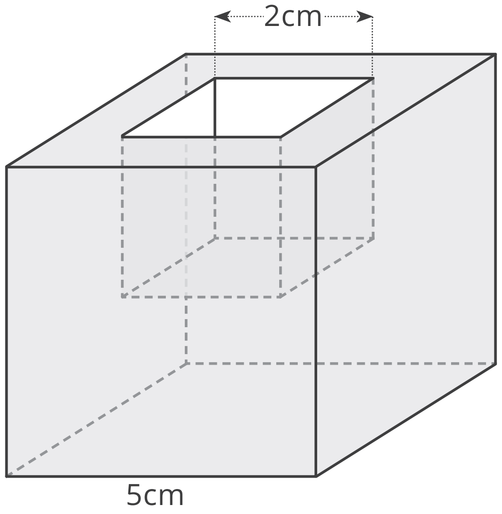

3. The surface area of the solid below is 108cm$^2$. Find the length of side $x$.




4. The solid below is formed of an isosceles triangular prism placed on a rectangular prism. Find its surface area.



5. The solid below is made up a cube removed from a larger cube. Find the volume and surface area of the shape.

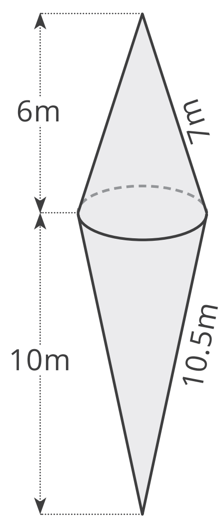



The following formulae are provided in GCSE exams, you may find them helpful for questions 6-8.

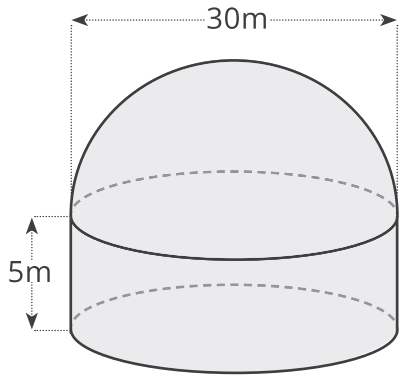
- Curved surface area of a cone = $\pi rl$
- Volume of a cone = $\frac{1}{3}\pi r^2 h$
- Volume of a pyramid = $\frac{1}{3}$ area of base × height (given in OCR exams only)
- Surface are of a sphere = $4\pi r^2$
- Volume of a sphere = $\frac{4}{3}\pi r^3$

6. Find the volume and surface area of each shape. Where possible, give your answers in terms of $\pi$.

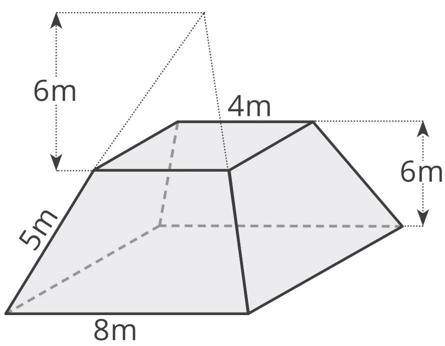
a. The shape below is made up of two cones. Each cone has a diameter of 6cm.



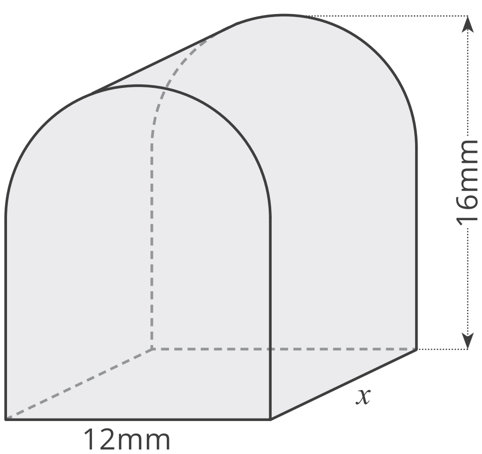
b. The solid below is made up of a cylinder topped with a hemisphere.



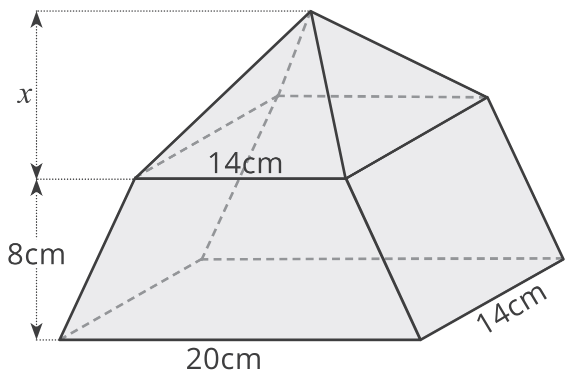

c. The is a square-based pyramid, with the top 6m removed.




7. The solid below is made up of a semi-circular prism placed on top of a rectangular prism. If its volume is (600 + 90π) $mm^2$, what is the length of side $x$?



8. The solid below is made up of a square-based pyramid placed on top of a trapezoid prism. The volume of the solid is $2296\text{cm}^3$. Find the length of side $x$.




**Challenge**

A machine part is made by taking a cube of metal of volume $512\text{cm}^3$ and removing a hemisphere from one face. The hemisphere has a diameter equal to the width of the cube.

The company is trying to reduce waste.

a. How many parts would they need to make for the waste material to be equivalent to one complete part?

b. The company does not want to change their manufacturing process. Using their current method, how many complete parts would they need to make for the waste to be sufficient to manufacture one new part?